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The Role of Granular Mechanics and Porous Flow for Ice Sheet Behavior in a Changing Climate

Anders Damsgaard, Jenny Suckale, Liran Goren

<https://adamsgaard.dk> <gopher://adamsgaard.dk> <anders@adamsgaard.dk>

ESCO 2020: Climate MS, 2020-06-08

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Introduction
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○○○○○○○○○ | Canadian Space Agency (CSA)
| Cocoates Limited [\(M](#page-13-0)[D](#page-14-0)A)
| Cocoates Limited (MDA)

Validation 0000

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 $A = \frac{1}{2}$, with geographic names discussed in the text. Pixel spacing in the text. Projection is 300

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Inter-Sediment simulation [Conclusions](#page-34-0)
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Subglacial sediment transport

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Subglacial sediment transport

Clark et al. 2018 Earth Surf. Process. Landforms

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Grounding-zone wedges

Anandakrishnan et al. 2007 Science

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Palaeo-grounding zone wedges

Bart et al. 2017 Scientific Reports

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Palaeo-grounding zone wedges

Bart et al. 2017 Scientific Reports

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Palaeo-grounding zone wedges

Bart et al. 2017 Scientific Reports

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Palaeo-grounding zone wedges

Dowdeswell et al. 2020 Science

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Ice-stream stabilization

Alley et al. 2007 Science

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Subglacial sediment transport

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No model for till transport ⇓ No physically-based modeling

Granular modeling

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Particle-scale modeling: Discrete-element method

Damsgaard et al. 2016 Geophys. Res. Lett.

Fixed $v_{\text{p,top}}^x$ Static, impermeable wall

Damsgaard et al. 2015 The Cryosphere

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 p_f low

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Particle-scale modeling: Discrete-element method

sphere

<git://adamsgaard.dk/sphere> C++, Nvidia C, cmake, Python, Paraview massively parallel, GPGPU detailed physics and fluid-grain coupling 20,191 LOC 3 months on nvidia tesla k40

 m^2q Particle 1 $m^{\perp}g$

Particle 2

Damsgaard et al. 2015 The Cryosphere

 p_f high

Continuum modeling of granular mechanics

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Phase transitions in granular materials

Houssais et al. 2015 Nat. Comm.

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Mohr Coulomb

Iverson 2010 J. Glaciol.

Charles-Augustin de Coulomb, b. 1736

Christian Otto Mohr, b. 1835

Karl von Terzaghi, b. 1883

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Local rheology for dense granular flows: *µ*(I), *Φ*(I)

$$
I = \frac{\dot{\gamma}d}{\sqrt{N/\rho}}
$$

Jop et al. 2005 J. Fluid Mech.:

$$
\tau = \mu(I)N
$$

$$
\mu(I) = \mu_{\rm s} + \frac{\mu_2 - \mu_{\rm s}}{I_0/I + 1}
$$

Pouliquen et al. 2006 J. Stat. Mech.:

$$
\Phi(I)=\Phi_{\text{max}}-(\Phi_{\text{max}}-\Phi_{\text{min}})I
$$

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 $\dot{\gamma} = g(\mu, N)\mu$

 $\xi(\mu) = \frac{Ad}{\sqrt{|\mu - \mu_{\rm s}|}}$

 $\frac{1}{\xi^2(\mu)}(g-g_{\text{local}})$

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A Non-local granular fluidity rheology 0 g $G\downarrow$ $v_\theta/(\Omega R_{\rm i})$ 0.6 perfect slip 0.2 $g_{\text{local}}(\mu, N) = \begin{cases} \sqrt{d^2 N/\rho_s}(\mu - \mu_s)/(b\mu) & \text{if } \mu > \mu_s \ 0 & \text{if } \mu < \mu_s \end{cases}$ $R_1/d = 68$ Ŧõ $(r - R_i)/d$ 0 if $\mu \leq \mu_s$ B 0.8 $\sum_{\substack{z \\ z > 0.4}}^{10.6}$ G 0.2 $\frac{\phi \rho_{\text{s}} G d}{P_{\text{wall}}}$ $= 0.145$ $\overline{2}$ z/d

Henann and Kamrin 2013 PNAS

 $\nabla^2 g = \frac{1}{526}$

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CNGF-PF: Cohesive NGF w. pore fluid

$$
\sigma'_{n}=\sigma_{n}-\rho_{f}
$$

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Model setup

Damsgaard et al. In review

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Validation

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Mohr Coulomb

Damsgaard et al. In review

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Strain distribution

Damsgaard et al. In review

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Granular modeling: Discrete-element method vs. continuum model

sphere

<git://adamsgaard.dk/sphere> C++, Nvidia C, cmake, Python, Paraview massively parallel, GPGPU detailed physics and fluid-grain coupling 20,191 LOC

3 months on nvidia tesla k40

1d-fd-simple-shear

git://adamsgaard.dk/1d_fd_simple_shear C99, makefiles, gnuplot single threaded simple physics, simple fluid-grain coupling 2,348 LOC 70 ms on 2012 laptop

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Water-sediment simulations

Specific sediment flux [m²/s]

Damsgaard et al. In review

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Water-sediment simulations

Deep or shallow deformation?

$$
d_{\rm s}=\sqrt{\frac{k}{\phi\,\eta_{\rm f}\beta_{\rm f}\pi f}}
$$

Damsgaard et al. In review

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Deep or shallow deformation?

Damsgaard et al. In review

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Deep or shallow deformation?

Maximum deformation depth: z'

$$
0 = \sqrt{2} \sin\left(\frac{7\pi}{4} - \frac{z'}{d_s}\right) + \frac{(\rho_s - \rho_f)Gd_s}{A_f} \exp\left(\frac{z'}{d_s}\right)
$$

Damsgaard et al. In review

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Water-sediment simulations

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Next steps: Ice-water-sediment coupling

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Conclusions

- First-principles granular rheologies promising for coupled simulations
- Rheology consistent with critical-state sediment mechanics and laboratory experiments
- Computationally lightweight compared to particle-based methods
- Towards testable field predictions of subglacial deformation and glacier dynamics

Resources

Slides:

<https://adamsgaard.dk/npub/esco2020-damsgaard.pdf>

Source code:

https://src.adamsgaard.dk/1d fd simple shear

Preprint: "Evolving basal slip under glaciers and ice streams" <https://arxiv.org/abs/2002.02436>